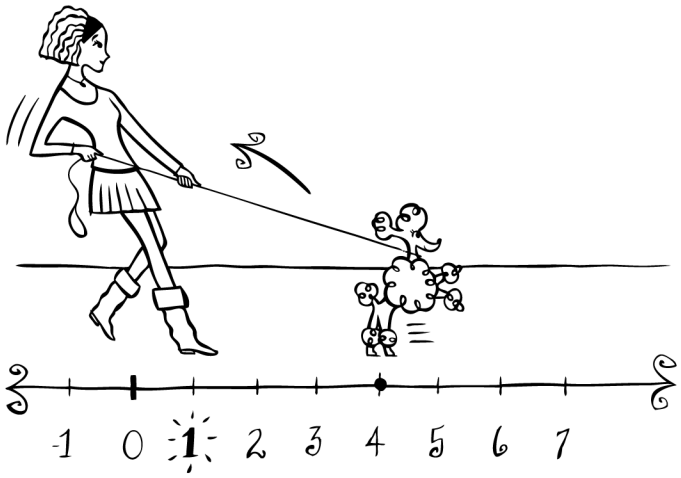
Sparky and the Meaning of Subtracting Negatives

Here's a wacky, totally different way to think about subtracting negatives.
(See the "What's the Deal" on p.48 of *Kiss My Math* for an easier way of thinking of it, but which requires understanding multiplication of negative numbers.)
On p.11 in *Kiss My Math*, I told you that we can only think about combining integers as walking up and down the number line when we have *addition* between the terms. I recommend always changing subtraction into "adding a negative," so that should take care of it, right? Not quite...How about a problem like 4 - (-3) = ?
On p.11, I showed you the shortcut, where the two negative signs cancel to make a postive + sign, but why does that work?
How can we *think about* subtracting a negative number? What does that even mean?
Let's pretend we don't have that shortcut, and see how we can use the number line after all.
Don't say I didn't warn you, though - it's pretty wacky!

Let's go to the number line. For the moment, I want you to think of numbers not as "things" but as "movements." So when you see "3" or "+3", it means "move in the positive direction 3 steps" and if you see "–3" it means "move in the negative direction 3 steps." And do you remember using the phrase "take away" to mean "subtract" in elementary school? We're going to use it again! Consider the normal subtraction problem: 4 – 3 = 1. In other words, 4 "take away" 3, equals 1.



Imagine you're walking your dog Sparky up the number line, in the positive direction, and he's gotten to 4, but then you pull *back* on his leash, moving him 3 steps in the negative direction, so he ends up at the "1." That's 4 – 3 = 1.

Now, if he had simply walked, on his own, from 4 to 1, then you might say he walked in the negative direction 3 steps. We could write that in math: 4 + (–3) = 1. But it's a sad story for Sparky, because he really wanted

to chase a squirrel that was hanging out somewhere around 10. Sparky was about to start walking in the positive direction, towards the squirrel, and when you gently pulled back on his leash to protect the squirrel, you were taking something away from Sparky. From his perspective, you were "taking away" his positive movement from 1 to 4. To Sparky, this felt more like subtraction: 4 – (+3) = 1 So, emotionally speaking, there are two ways to think about Sparky's movement. Mathematically, we can represent these two different perspectives of the same movement:

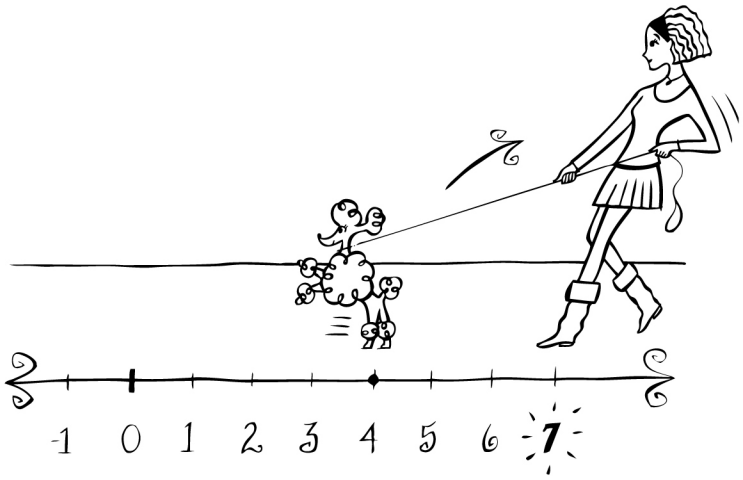
$$4 + (-3) = 1 \quad \text{vs.} \quad 4 - (+3) = 1$$

walking → ↑ in a negative direction; taking away ↗; positive movement ↖

So: Walking in the negative direction is the same as "taking away" movement in the positive direction.

I know that was a lot of fuss over the simple problem 4 – 3 = 1, but it'll help us with this next one!

Now, let's consider a slightly different situation. Now let's say Sparky is standing on the 4, but this time he's facing the negative direction –he just saw a mailbox he wants to pee on, over by the origin. He's about to start walking in the negative direction, and you want to stop him from even *thinking* about it. To do this, you gently pull Sparky's leash back 3 steps, and he ends up at the 7.



Again, there are two ways to think about Sparky's movement. If Sparky had willingly walked from the 4 to the 7, he would have *walked in the positive direction 3 steps*. That sounds like the addition problem: 4 + 3 = 7.

But from Sparky's point of view, something was taken away from him: his negative progress from 7 to 4! He wanted to move in the negative direction, and you stopped him from doing it. So, you took away negative movement. That's the subtraction problem: 4 – (–3) = 7, isn't it? Mathematically, again we can represent these two different perspectives of the same movement:

$$4 + 3 = 7 \text{ vs. } 4 - (-3) = 7$$

walking — in a positive direction — taking away — negative movement

So, oddly enough, ***taking away* movement in the negative direction is the same thing as walking in the positive direction.** In other words, *subtracting a negative number is the same as adding a positive number.*

By the way, this stuff can be tough to wrap your mind around! Try reading this Sparky example again after you've read Chapter 3 in Kiss My Math, and let it settle in your brain over time...

And here's the Shortcut from p.11:

Shortcut Alert: Subtracting Negative Numbers

When we are asked to *subtract a negative number* like this: 10 – (–5), we can just change the two negatives into a positive plus sign! So: 10 – (–5) 10 + 5 = 15. Now that's what I call a shortcut.

When the minus sign and the negative sign are next to each other like that, it's almost like one of them turns on its side and moves over until they're touching. Together they create the shape "+". It's a very special moment for the two of them.